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Analysis of Thermal Effects Associated with High-Energy Radiation in Reducing Scintillation of a Coaxial Beam

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The purpose of this theoretical study is to examine atmospheric effects on an optical propagation path using two coaxial beams, one a high-energy beam intended to modify the atmospheric channel for the other optical beam. The high-energy beam creates local heating of the atmosphere that tends to reduce the effective structure parameter of the refractive index, thereby reducing some of the deleterious effects of turbulence-induced scintillation. For the high-energy beam we considered millimeter waves (63 and 183 GHz) and an infrared wave (10.6 μm) propagating through extended atmospheric turbulence and a thin random-phase screen. We found that the use of a coaxial high-power millimeter beam with an optical beam can reduce the effective refractive-index structure parameter C_n^2 *for the low-power optical beam in the atmospheric channel, but mostly for the initial portion of the propagation path. Greater reduction in* C_n^2 *can be realized with the use of an infrared wave (10.6 μm) in place of a millimeter wave, because the absorption is greater than that of the millimeter beam. Overall, the most favorable atmospheric channel for the use of such coaxial beams appears to be one in which the atmospheric effects are limited primarily to a thin turbulent layer (phase screen) between the transmitter and receiver.*

KEYWORDS: Atmospheric optics, High-energy beam, Scintillation

Nomenclature

C_n^2	refractive index structure constant
$\hat{C}_n^2$	effective structure constant for extended turbulence
C_T^2	temperature structure constant
c_{nps}^2	effective structure constant for phase screen

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f	frequency
I, I_0	beam intensity
K	Gladstone–Dale constant
k	wave number (2/)
L	total propagation path length
L_1	distance from transmitter to phase screen
L_2	thickness of phase screen
L_3	distance from phase screen to receiver
n, n_0	index of refraction
P, P_0	power in beam
p_0	ambient pressure
$s(\mathbf{r})$	condensation
T	temperature in degrees Kelvin
t_0	heating time
v	wind velocity
W	beam radius at receiver
W_0	beam radius at transmitter
α	absorption coefficient
$\alpha_O(f)$	oxygen absorption
$\alpha_W(f)$	water vapor absorption
β_0^2	Rytov variance (scintillation index) of spherical wave
γ	ratio of molar specific heats
λ	wavelength
ρ, ρ_0	density of air
σ_1^2	Rytov variance (scintillation index) for a plane wave
$\Phi_n(\kappa)$	power spectrum of refractive-index fluctuations
Ω	surface water vapor concentration

1. Introduction

Atmospheric propagation of high-power electromagnetic radiation, such as laser beams or longer wavelength millimeter waves, can lead to a variety of effects.[6,8] Of particular importance are the consequences of i) the linear absorption and scattering due to the molecular and aerosol constituents of the atmosphere, ii) the distortion and scintillation of the beam due to atmospheric turbulence, iii) the self-induced thermal blooming that results from the absorption of a small amount of the beam power, and iv) the strong attenuating effects of the plasmas resulting from gas breakdown at extreme high power.

The essential feature of thermal blooming concerning an electromagnetic wave propagating through the atmosphere involves the treatment of the atmosphere as an ideal gas having some absorption at the wavelength of the propagating wave. This absorption will give rise to a local heating that in turn will produce a small pressure increase. The atmosphere then expands (at the speed of sound) in order to restore pressure balance, leaving behind a small decrease in the density. The local refractive index will decrease relative to the surrounding medium in proportion to the local density change, eventually resulting in an increase in diameter of the entire beam known as thermal blooming.[4,6,8] The net effect is a spreading or distorting shape of the mean intensity profile of a beam.

Other studies concerning thermal blooming effects have concentrated on the intensity fluctuations of the high-energy beam. For example, Agrovskii et al.[1,2] experimentally and theoretically examined intensity fluctuations of a beam wave. They found that such fluctuations are initially weakened under an increase in transmitted power (several watts per square centimeter) up to some critical energy density value and later increased as the energy density exceeds this value.

In our analysis we consider the local heating associated with various millimeter and infrared (IR) beams and its effect on reducing the local structure constant C_n^2 of the refractive index. Our investigation in each case is based on the dual propagation of continuous wave (CW) coaxial beams through the same channel. The objective of this study is to determine the feasibility of using high-energy millimeter or IR beams on "conditioning the propagation channel" so as to reduce the local structure constant and, consequently, the effects of scintillation on a low-power dual coaxial propagating laser beam. Such schemes may be useful for propagation links as found in particular laser communication networks and some links for laser radar and directed energy.

2. Thermal Conditioning of the Channel

To begin, we review some basic ideas associated with beam absorption and then draw on those aspects that are necessary to predict the associated temperature changes. The subject of beam heating of the propagation path draws heavily on certain aspects of fluid mechanics, on wave propagation in fluids, and on both geometrical and physical optics. To achieve a simple order-of-magnitude estimate for beam heating in an ideal gas, we consider an unfocused CW Gaussian beam of total power P being emitted through an aperture of radius W_0 into air at 1-atm pressure. After the beam is turned on and in steady state, the refractive index will vary with time because of heating produced by the beam.

The "uniform" heating of the propagation channel is associated with the distance defining the Rayleigh range, which we determine from the relation $\lambda L/\pi W_0^2 = 1$, or $L = \pi W_0^2/\lambda$, where L is the propagation distance, W_0 is the initial beam radius, and λ is the wavelength. At millimeter wavelengths this corresponds to only a few meters, but at IR wavelengths it might be more than 100 m.

Over a wide range of values for the temperature, pressure, and electric field, we find that in air the refractive index n can be expressed by the relation

$$n = 1 + K\rho, \tag{1}$$

where ρ is the density of air and K is the Gladstone–Dale constant. If the density varies by a small amount $\delta\rho$, the corresponding change in refractive index can be deduced from Eq. (1); that is,

$$\frac{\delta n}{n_0 - 1} = \frac{\delta\rho}{\rho_0} = s, \tag{2}$$

or

$$\delta n = (n_0 - 1)s, \tag{3}$$

where ρ_0 and n_0 are the initial quiescent values and s is the condensation. It has been shown that the condensation s at an arbitrary point $\mathbf{r}$ in the beam is given by[8]

$$s(\mathbf{r}) = -\frac{\gamma - 1}{\gamma p_0}\alpha I(\mathbf{r})t_0. \tag{4}$$

Here, p_0 is the ambient pressure, γ is the ratio of the molar specific heats c_p and c_V, α (m^{-1}) is the beam absorption coefficient, I is the beam intensity at the point in question, and t_0 is the heating time.

Equation (4) is based on no air movement through the beam and that the intensity at the point in question remains constant during the heating time t_0. Let us consider the case in which there is a flow of air moving through the beam (now of diameter $2W$) at velocity v normal to the axis of the beam, leading to $t_0 = 2W/v$. By assuming that the power in the beam is (accounting for attenuation)

$$P = \tfrac{1}{2}\pi W_0^2 e^{-\alpha L} = P_0 e^{-\alpha L}, \tag{5}$$

and taking the intensity $I(\mathbf{r})$ of the beam at any location along the propagation path to be uniform across an area defined by the beam's spot size, i.e., $I_0 = 2P/\pi W^2$, the temperature change T due to beam energy absorption can be estimated from the expression

$$\frac{\delta T}{T} = -s = \left(\frac{\gamma - 1}{\gamma p_0}\right)\frac{4\alpha P}{\pi v W}, \tag{6}$$

where T is the temperature in degrees Kelvin. Hence, we see that the temperature change is

$$\delta T(^\circ T) = -sT = \left(\frac{\gamma - 1}{\gamma p_0}\right)\frac{4\alpha TP}{\pi v W}. \tag{7}$$

Typical values of the wind velocity are $v = 1$–3 m/s, and parameters in Eq. (7) for a standard atmosphere are the following (which are essentially fixed):

$$p_0 = 1{,}013 \text{ mbar}$$
$$\gamma = 1.4$$

2.1. Refractive index fluctuations

The index of refraction structure parameter C_n^2 and temperature T in degrees Kelvin are related by the well-known formula[7]

$$C_n^2 \approx \left(\frac{79 \times 10^{-6} p_0}{T^2}\right)^2 C_T^2, \tag{8}$$

where C_T^2 is the temperature structure constant. Based on Eq. (8) and the model for the change in temperature δT given by Eq. (7), the implied or effective refractive-index structure constant $\hat{C}_n^2$ associated with the heated atmosphere is approximately

$$\hat{C}_n^2 \approx \left[\frac{79 \times 10^{-6} p_0}{(T + \delta T)^2}\right]^2, \qquad C_T^2 = \left(\frac{T}{T + \delta T}\right)^4 C_n^2. \tag{9}$$

From Eqs. (9) it is clear that any small positive change in temperature δT of the ambient medium [see Eq. (7)] will lead to a reduced effective structure constant.

2.2. Scintillation by atmospheric turbulence

Scintillation of the coaxial optical beam wave simultaneously transmitted with the high-power beam refers to small fluctuations in its received intensity caused by refractive-index fluctuations. In general, scintillation depends on beam characteristics of the transmitted wave and on three atmospheric parameters: inner scale l_0, outer scale L_0, and C_n^2. However, we can discuss scintillation effects quite simply in terms of the simple spherical wave model for the beam wave using the basic Kolmogorov spectrum

$$\Phi(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \tag{10}$$

where κ is the atmospheric wave number. For the purposes of this study, weak fluctuation theory is also adequate to describe scintillation. Under weak fluctuations, the scintillation index of a spherical wave based on the Kolmogorov spectrum model (10) is given by

$$\beta_0^2 = 0.5 C_n^2 k^{7/6} L^{11/6}, \tag{11}$$

where $k = 2\pi/\lambda$ is the optical wave number. Consequently, any reduction realized in the structure constant C_n^2 will directly lead to a reduction in β_0^2.

3. Millimeter Beam Waves

The term *millimeter wave* generally refers to that portion of the electromagnetic spectrum between 30 and 300 GHz. This corresponds to wavelengths varying between 10 and 1 mm; hence, millimeter waves lie between the microwave and IR portions of the spectrum. Interest in the use of millimeter waves has been up and down over the decades, starting as early as the 1940s. The three principle characteristics of millimeter waves—short wavelengths, large bandwidth, and interaction with atmospheric constituents—can be viewed as both advantages and disadvantages, depending on a particular application.

3.1. Absorption by gases and water vapor

In general, the attenuation of millimeter waves in a clear atmosphere near ground level is negligible for most practical purposes except around frequencies where the absorption lines of water vapor or oxygen are situated. This means that attenuation will be highest around 60, 119, and 183 GHz, which correspond to oxygen molecule (first resonance), oxygen molecule (second resonance), and water vapor (third resonance) absorption, respectively. Besides the molecular absorption due to atmospheric gases and water vapor, the attenuation due to other factors, such as rain and fog, is even more severe. In addition, there is the effect of index of refraction fluctuations (although much weaker) similar to those associated with the shorter wavelength optical waves.

For a terrestrial path near the ground, the path attenuation may be expressed as

$$A(f) = \alpha_O(f) + \alpha_W(f) \text{ (dB/km)} \tag{12}$$

where $\alpha_O(f)$ is the oxygen absorption and $\alpha_W(f)$ is water vapor absorption, both dependent on the operating frequency f expressed in gigahertz. For frequencies $f > 63$ GHz, the path attenuation due to oxygen absorption near sea level can be approximated by[5]

$$\alpha_O(f) \approx \left[3.79 \times 10^{-7} f + \frac{0.265}{(f-63)^2 + 1.59} + \frac{0.028}{(f-118)^2 + 1.47}\right] \times (f+198)^2 \times 10^{-3} \text{ (dB/km)}. \tag{13}$$

Note that the resonances of oxygen at 63 and 118 GHz are modeled as poles in the donominator of Eq. (13). The approximation for water vapor at sea level with a temperature of

15°C is given by

$$\alpha_W(f) \approx \left[0.050 + 0.0021\Omega + \frac{3.6}{(f-22.2)^2+8.5} + \frac{10.6}{(f-183.3)^2+9} + \frac{8.9}{(f-325.4)^2+26.3}\right] f^2\Omega \times 10^{-4}\,(\text{dB/km}), \tag{14}$$

where Ω is the surface water vapor concentration in grams per cubic meter. This expression is considered valid for $f < 350$ GHz and $\Omega < 12$ g/m^3. Last, the absorption coefficient α is related to the attenuation A in decibels per kilometer by the relation

$$\alpha = \frac{A(f)}{4.34 \times 10^3}\,(\text{m}^{-1}). \tag{15}$$

We calculated the change in effective structure parameter $\hat{C}_n^2$ for different microwave frequencies as predicted by Eq. (9) up to a total propagation path of 200 m. We found that the resulting effective structure constant $\hat{C}_n^2$ was always lower (but less than an order of magnitude) near the transmitter over the first 20 m or so, after which it asymptotically approached the original value C_n^2 with increasing propagation distance. Lower wind speeds are more favorable to decreasing the structure constant, as are higher frequencies (183.3 GHz) because of the increased absorption.

4. Optical Beam Waves

The optical portion of the electromagnetic spectrum extends roughly over wavelengths from 10^{-2} to 10^3 μm, which includes the ultraviolet (UV), visible and IR wave bands. Atmospheric absorption is very wavelength sensitive, as mentioned above. Below about 0.2 μm, absorption by O_2 and O_3 is so strong that there is essentially no propagation. This spectral region is known as the vacuum UV because propagation at these wavelengths is virtually impossible except in a vacuum. On the other hand, there is virtually no absorption at the visible wavelengths (0.4–0.7 μm) out to roughly 1.3 μm.

Absorption windows for good transmittance above 1.3 μm exist between 1.5 and 1.7 μm and between 2.0 and 2.5 μm. The major thermal imaging IR absorption windows are between 3.4 and 4.2, 4.5 and 5.0, and 8.0 and 13 μm. Between these absorption windows are absorption bands where transmission is limited or essentially negligible.

If we consider a CO_2 laser operating at 10.6 μm and assume that the transmittance at 1 km is on the order of 65–70% of transmitted power, the implied exponential coefficient is roughly $\alpha = 0.0004$. For the purpose of the following analysis we will make this assumption.

In Fig. 1 we repeat the analysis described in the preceding section, except that here we replace the millimeter wave with an optical wave at 10.6 μm. We assume that the refractive-index structure parameter is 10^{-13} m$^{-2/3}$, the power in the optical wave at the transmitter is $P_0 = 2{,}000$ W, the beam radius at the transmitter is 4 cm, and the beam is focused at 200 m. Focusing the beam leads to lower values of the effective structure constant $\hat{C}_n^2$ along the path between 120 and 200 m, as illustrated in Fig. 1. Average wind speeds of $v = 1$ and 3 m/s were considered for this analysis. The reduction in effective structure constant $\hat{C}_n^2$ in this case is greater than that for the millimeter wave case, and the reduction is observed to be greater for the lower average wind speed. However, this is still a modest reduction in effective structure constant, even at low wind speeds.

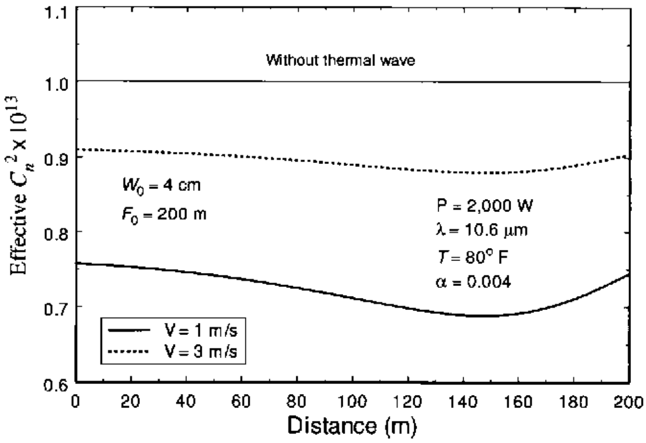

Fig. 1. Scaled effective structure parameter vs. propagation distance for an optical wave with initial power $P_0 = 2{,}000$ W and wavelength 10.6 μm. The initial diameter of the optical wave is 8 cm, and the beam is focused at 200 m.

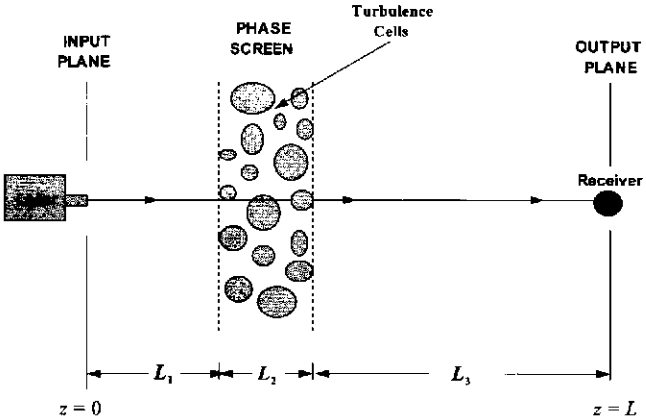

Fig. 2. Propagation geometry for a random-phase screen.

5. Phase Screen Analysis

The notion of a phase screen has been used over the years for a variety of physical situations.[3] A phase screen is ordinarily treated as a slab of random medium located somewhere between source and receiver. It may naturally exist in the boundary layer around an aircraft, around buildings, near hot exhaust from a vehicle, or possibly above a ground fire confined to a small area. Any small atmospheric layer in which there exist significant temperature differentials can be modeled as some type of "phase screen."

Let us consider the geometry illustrated in Fig. 2 in which a laser transmitter is located in the plane at $z = 0$ and propagation is along the positive z axis. We assume that the distance between $z = 0$ and L_1 and that between $z = L_1 + L_2$ and $L_1 + L_2 + L_3$ are essentially free of atmospheric turbulence effects. Between $z = L_1$ and $z = L_1 + L_2$, however, is a short distance where optical turbulence is assumed to exist. We refer to this short distance as a "thin" phase screen provided that the condition $L_2/L_3 \ll 1$ is satisfied. Under this last condition, only the phase of the optical wave is initially distorted as the optical wave emerges from the screen. As the optical wave propagates some distance after leaving the screen, amplitude fluctuations will also begin to develop and steadily increase in much the same fashion as propagation through extended turbulence.

The Rytov variance for an optical wave propagating through a thin random-phase screen as illustrated in Fig. 2 is known to be given by the expression[3]

$$\sigma_1^2 = 2.25c_{\mathrm{nps}}^2 k^{7/6} L_3^{5/6} L_2, \tag{16}$$

where c_{nps}^2 is the effective refractive-index structure constant for the phase screen. Typical values for this structure constant may be on the order of 10^{-10} m$^{-2/3}$, often much higher than that for an extended medium. Local heating of the path through the phase screen may be accomplished in the same manner as described above for an extended medium by propagating a high-power millimeter wave. The reduction in the effective structure constant for the phase screen [similar to that in Eq. (9)] can lead to reduced scintillations in an optical wave that is coaxially transmitted as before.

5.1. Graphical analysis

In Fig. 3 we show the implied Rytov variance associated with a collimated Gaussian-beam wave at wavelength 1.06 μm within the presence of a coaxial millimeter beam wave at frequency $f = 183$ GHz. The radius of both beams was taken to be 2 cm, the refractive-index structure parameter for the phase screen was initially 10^{-10} m$^{-2/3}$, and the power in the millimeter wave at the transmitter was $P_0 = 1{,}000$ W. The solid curve corresponds to an average wind speed through the phase screen of 1 m/s, the dashed curve corresponds to an average wind speed of 3 m/s, and the dashed–dotted curve corresponds to the absence of the millimeter wave. The phase screen was assumed to be situated directly in front of the transmitter in all three cases.

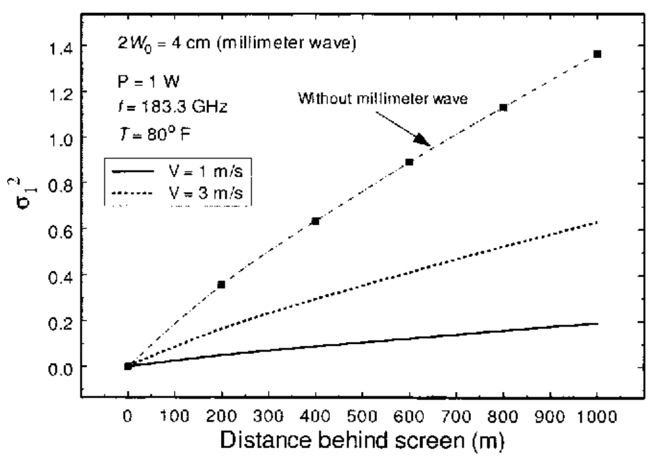

Fig. 3. Scaled Rytov variance for a collimated Gaussian-beam wave vs. propagation distance for a millimeter wave with initial power $P_0 = 1{,}000$ W and frequency $f = 183$ GHz. The initial diameter of both beams is 4 cm. The phase screen is placed close to the transmitter.

Note that the drop in Rytov variance corresponding to low average wind speeds (~1 m/s) is typically by a factor of four, and even that for higher average wind speed (~3 m/s) is a factor of two. By moving the phase screen to different locations along the path, the implied Rytov variance will change in all three cases, but the relative difference between that with the millimeter wave and that without will remain essentially the same.

6. Discussion

In this study we introduced an order-of-magnitude model for atmospheric heating caused by the high power of a millimeter wave to study the feasibility of reducing scintillation of a coaxial optical wave along with the millimeter wave. This analysis led to an approximate expression for the temperature differential from which we could infer the change in effective index of refraction structure parameter. In arriving at our results, we made several simplifying assumptions, including that the temperature differential δT is uniform across the millimeter wave propagation path.

In general, we found that i) the propagation of a high-power millimeter wave through the atmosphere can reduce C_n^2 along the path in the portion of the propagation path, ii) higher frequency millimeter waves are more effective in reducing the structure constant C_n^2 than are lower frequency waves (63 GHz), and iii) lower average transverse wind speeds lead to a greater reduction in C_n^2. Also, given a comparable value for the attenuation coefficient α, replacing the millimeter wave with an optical wave of wavelength 10.6 μm can lead to a reduction in C_n^2 over a 200-m path, provided that the beam is focused at the receiver (see Fig. 1).

Because the reduction in C_n^2 occurs only near the transmitter, we believe that the use of a high-power coaxial millimeter wave with an optical wave can be most effective in reducing scintillation of the optical wave in propagation channels where atmospheric effects are confined primarily to a thin layer known as a phase screen (see Figs. 2 and 3). This might be the case, for example, when the transmitter is located in (or on top of) a tall building or on a moving aircraft. In either case there will exist a "turbulent boundary layer" that is formed by large temperature gradients next to the building or around the aircraft.

The analysis presented here is for a CW high-power beam and a coaxial beam. The reduction in the turbulence parameter C_n^2 might be more fully exploited by using a pulsing scheme. The initial pulse could be a "channel conditioning" pulse with a pulse period less than the time required for refractive changes in the medium. That would be a period less than the time for the density change in the channel that occurs at the speed of sound in the medium. Thus, the heating would not cause self-distortion of the "conditioning" pulse. The channel-conditioning beam could also be spatially shaped to more favorably condition the channel uniformly for the trailing pulse. The appropriately timed trailing pulse of the coaxial beam pulse would then "see" a reduced turbulent condition within its propagation path. Last, for a transverse wind or in a slewing scheme, the conditioning pulse beam and the transmitted pulse beam could be transversely separated so that the conditioned channel "blows" into alignment with the transmitted beam.

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