

# Low-Cost Experiment to Measure Optical Turbulence Between Two Buildings

Thomas Farrell,<sup>1</sup> David Dixon, Lee Heflinger, Stanley Klyza,  
and Kenneth Triebes\*

Northrop Grumman Space Technology, One Space Park,  
Redondo Beach, California 90278

Northrop Grumman Space Technology is developing adaptive optics (AO) technology for use in tactical laser weapons systems, as well as other applications. As part of this effort, we are currently building and operating an AO demonstration between the roofs of two buildings on our Space Park campus. To make preliminary AO design estimates, it was necessary to have accurate knowledge of the turbulence strength along the path. Since the campus sits in an urban area with many tall buildings and parking lots, where diurnal heating and local effects can be expected to dominate, predicting the turbulence strength analytically was not practical. Therefore, we devised a low-cost experiment that determines, in near real time, the turbulence strength along the path. Fundamentally, the experiment consists of a set of silvered spherical reflectors at the transmit location, each catching a glint from the sun and sending it to the receiver's location. At the receiver, the apparent tilt angles between pairs of spheres are measured as they change due to the atmospheric turbulence. The variance of these tilt angles is then used to find the turbulence strength. This use of differential tilt to determine turbulence strength is attractive because it is invulnerable to common motion effects due to building or receive telescope jitter. We describe, in detail, the principle of operation of this experiment and our setup. We then present results.

**KEYWORDS:** Atmospheric turbulence, Coherence length, Laser propagation, Scintillation, Turbulence measurement

## Nomenclature

$C_n^2$	index of refraction structure function, a measure of turbulence strength
$D$	diameter of the receive telescope aperture
$D_c$	diameter of the receive telescope central obscuration
$d(z)$	separation of the glint paths along the optical path (see Fig. 4)
$J_0(\bullet), J_1(\bullet)$	Bessel functions of the first kind, zero and first orders
$L$	total path length between O2 and R1, the two buildings at Space Park used for the experiment

---

Received March 23, 2006; revision received May 24, 2007.

\*Corresponding author; e-mail: ken.triebes@ngc.com.

<sup>1</sup>Currently at Starfire Optical Range, Kirtland Air Force Base, NM 87117; e-mail: thomas.farrell@kirtland.af.mil.

$N, i$	number of sections dividing the path, and an index from 1 to $N$ used as described in the text (see Fig. 4)
$Ry$	Rytov number
$r_0$	Fried coherence length
$z$	distance along the optical path, from the transmitter
$\sigma^2$	measured differential tilt variance between two spheres
$\sigma_\chi^2$	log-amplitude variance
$\kappa$	magnitude of the two-dimensional spatial frequency perpendicular to the optical path direction
$\lambda$	wavelength of interest

## 1. Introduction

Northrop Grumman Space Technology (NGST) has set up a 590-m laser beam path from the roof of building O2 to the roof of building R1 at Space Park (Fig. 1) and is building and running an adaptive optics (AO) system in a penthouse on the roof of R1 to correct wavefront aberration due to atmospheric turbulence.

To properly size the AO system, and to determine the expected performance, it was necessary to have some knowledge of the turbulence strength along the path. However, since the campus sits in an urban area with many tall buildings and parking lots, where diurnal heating and local effects may dominate, predicting the turbulence strength analytically, with any degree of confidence, was not practical.

It is also highly desirable to have a real-time estimate of the turbulence strength while the AO system is operating, to determine the expected level of performance and to aid in diagnosing problems.

We considered purchasing a commercial scintillometer for these purposes<sup>5</sup> but decided it would be more cost effective, and would serve our ultimate objective of better understanding the nature of the atmospheric turbulence in this specific location, if we built and used our own turbulence measurement equipment.

Others have reported similar experiments. For example, in Ref. 2 a commercial scintillometer is used to measure the turbulence strength over a 185-m-long path. In Ref. 3, the experimenters built their own system and used it to measure turbulence strength over a 16-km path over water. In both of these cases the turbulence strength was assumed to be constant along the entire path. In our experiments we make the same assumption, with reservations that are discussed below.

We decided on a two-part plan: 1) Prior to AO construction, we would conduct an inexpensive, relatively simple, experiment that would give us a good idea of the level of turbulence we might expect along the path. 2) As part of the AO design itself, we will use the laser light transmitted over the path to make real-time turbulence measurements while the AO system is in operation.

In this paper we explain the principle of operation of the experiment for part 1 of the plan, describe the setup and operation, and show the results we obtained.

## 2. Principle of Operation

The "transmitter" for the experiment is shown in Fig. 2. On the roof of building O2, we had 11 2-in. (5 cm)-diameter silvered spherical reflectors attached to a 7-ft (2.1 m)-long, almost 8-ft (2.4 m)-tall sawhorse. The spherical reflectors were nothing more than Christmas

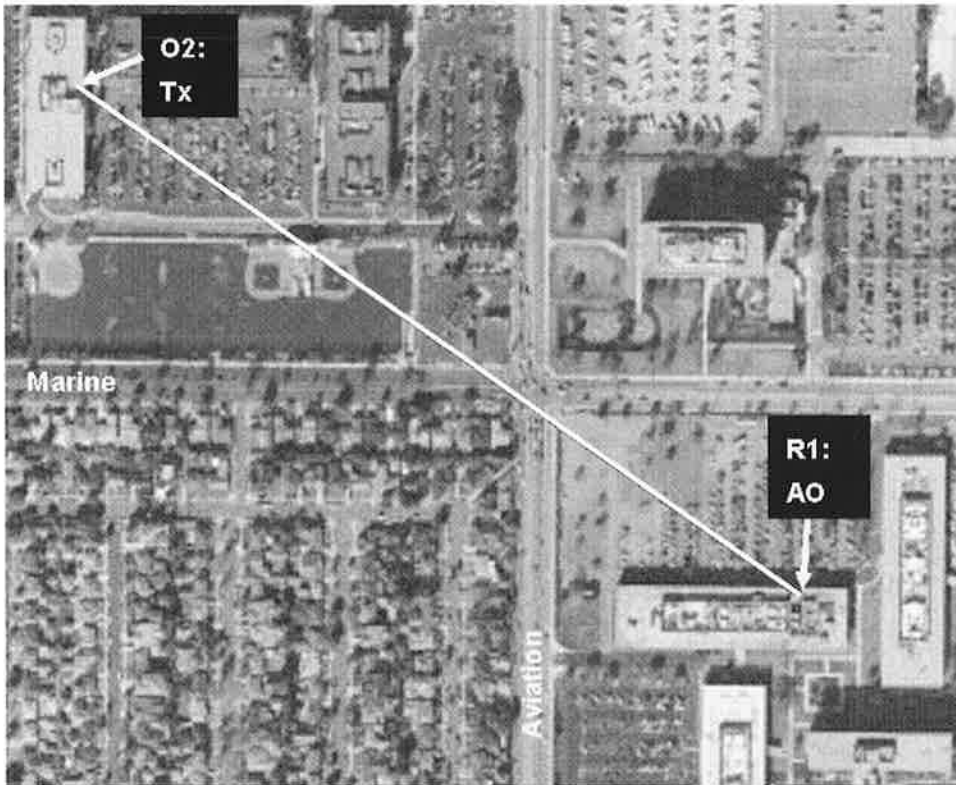


Fig. 1. Path of the AO demonstration at NGST’s facility at Space Park, Redondo Beach, California.

ornaments. Each catches a glint from the sun and sends it to the receive telescope set up in the penthouse of R1. Attached to the telescope is a digital camera set up to image the glints.

The receiver is shown in Fig. 3. It consisted of a 90-mm-diameter Meade telescope, with an EPIX, Inc., digital camera attached. The camera was connected to a personal computer. We used software that came with the camera, but used Matlab code to interface with the camera software, as well as to display, process, and save the data collected.

Both atmospheric turbulence and jitter due to building and telescope mount motion cause the glint images to “dance” on the focal plane. However, while all of the glint images tend to dance together from building and telescope mount jitter, there is a large component of differential motion from the atmospheric turbulence. Therefore, in this experiment, we captured a large number of camera frames over a short period of time and measured the distance between glint images on each. From these, we computed the tilt variance between each pair of glints.

By modifying an equation presented in Ref. 4, we were able to relate the measured tilt variance to the turbulence strength along the path:

$$C_n^2 = \frac{-D^2 \cdot [1 - (D_c/D)^2]^2 \cdot N}{41.7 \cdot L \cdot \sum_{i=1}^N \int_0^\infty k^{-8/3} \cdot \{J_1[(z_i/L) \cdot (\kappa D/2)] - (D_c/D) \cdot J_1[(z_i/L) \cdot (\kappa D_c/2)]\}^2 \cdot \{J_0[(z_i/L) \cdot \kappa \cdot d(z_i)] - 1\} \cdot d\kappa} \cdot \sigma^2 \quad (1)$$



**Fig. 2.** Spherical reflectors on wooden sawhorse on O2 roof. (The vertical spar that is partially shown in the photograph was removed prior to the experiment. Golden colored spheres were replaced with silver colored ones prior to the experiment.)

To obtain Eq. (1), we began with Eq. (7.55) in Ref. 4, derived by Sasiela to describe the differential Zernike tilt variance that would result from the light from two sources traveling along parallel paths through a turbulent medium. We modified the equation to account for 1) the converging glint paths to the single receive telescope aperture, 2) the central obscuration in the telescope, and 3) gradient tilt vs Zernike tilt.

The latter modification was made because, in our experiment, we measured the glint positions on the focal plane using a centroid calculation, which is better described by gradient tilt.

The resulting equation originally included a three-dimensional integral, two in spatial frequency perpendicular to the path and one in real space along the path. Sasiela shows that by taking the spatial frequency in polar coordinates, the integral over angle can be solved analytically. The integral over spatial frequency magnitude was solved numerically in our experiment.

The integral along the path was solved by breaking up the path into  $N$  sections, as shown in Fig. 4, and summing the results for each section. The distance between each of the glint paths, in each section, was approximated as a constant equal to the actual distance at the center of the section;  $N = 10$  was found to be an adequate number of sections for our purpose.



Fig. 3. 90-mm Meade telescope and EPIX camera in R1 penthouse window.

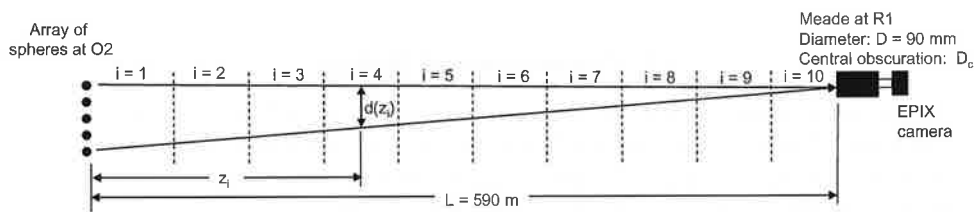


Fig. 4. Geometry of optical path.

We then assumed that the turbulence strength was constant along the path and inverted the results to obtain the index-of-refraction structure constant as a function of the measured differential tilt variance. The result is Eq. (1).

It may be questioned whether the turbulence strength can be assumed to be constant along the path, particularly since local turbulence effects may be supposed to be stronger near the two buildings. We readily admit that this is a shortcoming of this experiment. However, as

mentioned above, we note that a similar assumption is made for similar experiments. In our case, our primary goal is to determine the effects of turbulence on an AO system located at our receiver location. Therefore, we are less interested in where the turbulence originates than in how it affects the wavefront at the receiver.

We also note that Sasiela developed his equation assuming Kolmogorov turbulence. Indeed the  $C_n^2$  parameter itself is defined as a coefficient in the Kolmogorov turbulence spectrum.<sup>1</sup> In our case, we may well have less than fully developed Kolmogorov turbulence. However, as stated above, we are primarily interested in the practical effects of the turbulence on the AO system design and performance. (This is a major consideration for our next turbulence measurement experiment along this path, and is discussed in the final section of this paper.)

The Fried coherence length is an important parameter that may be derived from  $C_n^2$  (Ref. 1). For a spherical wave, assuming constant turbulence strength, it is

$$r_0 = 3.01 \cdot [C_n^2 \cdot L \cdot (2\pi/\lambda)^2]^{-3/5}. \quad (2)$$

Among other things, this parameter may be used to estimate the “fitting error,” a hit to AO performance due to wavefront sensor subaperture size and actuator spacing. Another important parameter that may be derived from  $C_n^2$  is something we call the Rytov number. For a spherical wave, assuming constant turbulence strength, it is computed as

$$Ry = 0.124 \cdot (2\pi/\lambda)^{7/6} \cdot L^{11/6} \cdot C_n^2. \quad (3)$$

For mild turbulence, this has been shown, both in experiments and in wave-optics simulations, to be nearly equal to the log-amplitude variance ( $\sigma_\chi^2$ ) of the wavefront at the receive aperture plane.<sup>1</sup>

In the results presented below, both the Fried coherence length and the Rytov number have been computed for  $\lambda = 640$  nm, and the turbulence along the path has been found to be mild. This indicates that an AO system with a Hartmann wavefront sensor should work well to correct the wavefront phase at the end of the path.

### 3. Experimental Setup

The transmitter and receiver are described above and shown in Figs. 2 and 3.

All images from the camera are 1,024 rows tall  $\times$  1,280 columns wide, but only a small portion of each image was used. Each pixel outputs a 10-bit number. With our setup, the pixel spatial resolution was found to equal 4.24  $\mu$ rad. Neglecting turbulence, adjacent spheres were separated by 356  $\mu$ rad, as seen from the telescope.

Even during the mildest periods, the standard deviation of the differential tilt was generally found to be several pixels, and so there was adequate resolution. The turbulence was never so strong that glints from adjacent spheres would intrude into a given sphere’s centroid calculation region. The centroid calculation region for each sphere was set to 75  $\times$  75 pixels.

One serious limit on this experiment was the “rolling integration” performed by the camera. Instead of integrating over the entire focal plane simultaneously, each horizontal line was integrated in turn. This required us to use a relatively short integration time (1 ms) to ensure that the camera lines in the centroid calculation regions were covered in a short

time interval, so that the effects of turbulence did not cause the glint to blur seriously while the frame was collected.

The camera readout time limited our ability to take more than about one frame per second and so prevented us from taking meaningful high-speed temporal data with this experiment.

#### 4. Operation

Upon initial startup, the Matlab program instructed the EPIX software to take a single image with an integration time of 20 ms. The EPIX software saved this image to a file that was read by the Matlab program. Matlab displayed the image on a figure window.

The operator would put the cursor over each glint in the image, clicking to set the center of each glint's centroid calculation region. The operator then instructed the program to continue. At that time, the Matlab program ran in an indefinite loop until the operator pressed the "Interrupt" button.

In each cycle of the loop, the EPIX software was instructed to take 100 frames at one frame per second, each with an integration time of 1 ms, and to save the frames to files. The Matlab program read each file in turn, throwing away pixel counts lower than 200 (of a maximum of  $2^{10} = 1,024$  at camera pixel saturation) to eliminate background light and focal plane noise, and computed the centroid positions of each of the glints. In some cases, particularly at night and during cloudy periods, no pixels in a centroid region would be found with more than 200 counts, and that glint was not used in the calculations described below. The program also threw out glints that were so bright that some of the pixels in the centroid calculation region were saturated.

After doing this for all 100 frames, the Matlab program computed the mean distance and the tilt angle variance for each pair of glints and used Eq. (1) to estimate  $C_n^2$ . The values of  $C_n^2$  for each pair of glints were then averaged to find a single  $C_n^2$  value for the cycle. The average position of each glint on the focal plane was used to compute new centroid calculation regions for each glint for the next cycle.

The program used the computed average value of  $C_n^2$  to estimate the Fried coherence length and the Rytov number, using Eqs. (2) and (3). These values were displayed on the screen and also saved to files.

Each cycle of the indefinite loop took approximately 5.5 min.

By taking frames at 1-s intervals, we believe we obtained images whose differential tilts due to atmospheric turbulence were statistically independent from frame to frame. By reducing the exposure time as much as possible (to 1 ms), we believe that we caught nearly instantaneous images of each glint, with little blurring due to turbulence or errors due to the rolling integration of the camera.

#### 5. Results

The experiment was conducted from November 25 through December 22, 2004.

Figure 5 shows the mean power levels (in counts on the focal plane) measured from the glints during the experiment. The results agree well with observations of fog and overcast. In the data shown in Figs. 6–9, some strange-looking results may actually be the result of too low power to make adequate calculations. The reader is cautioned to check with Fig. 5 to ensure that the glints were sufficiently bright for the periods in question.

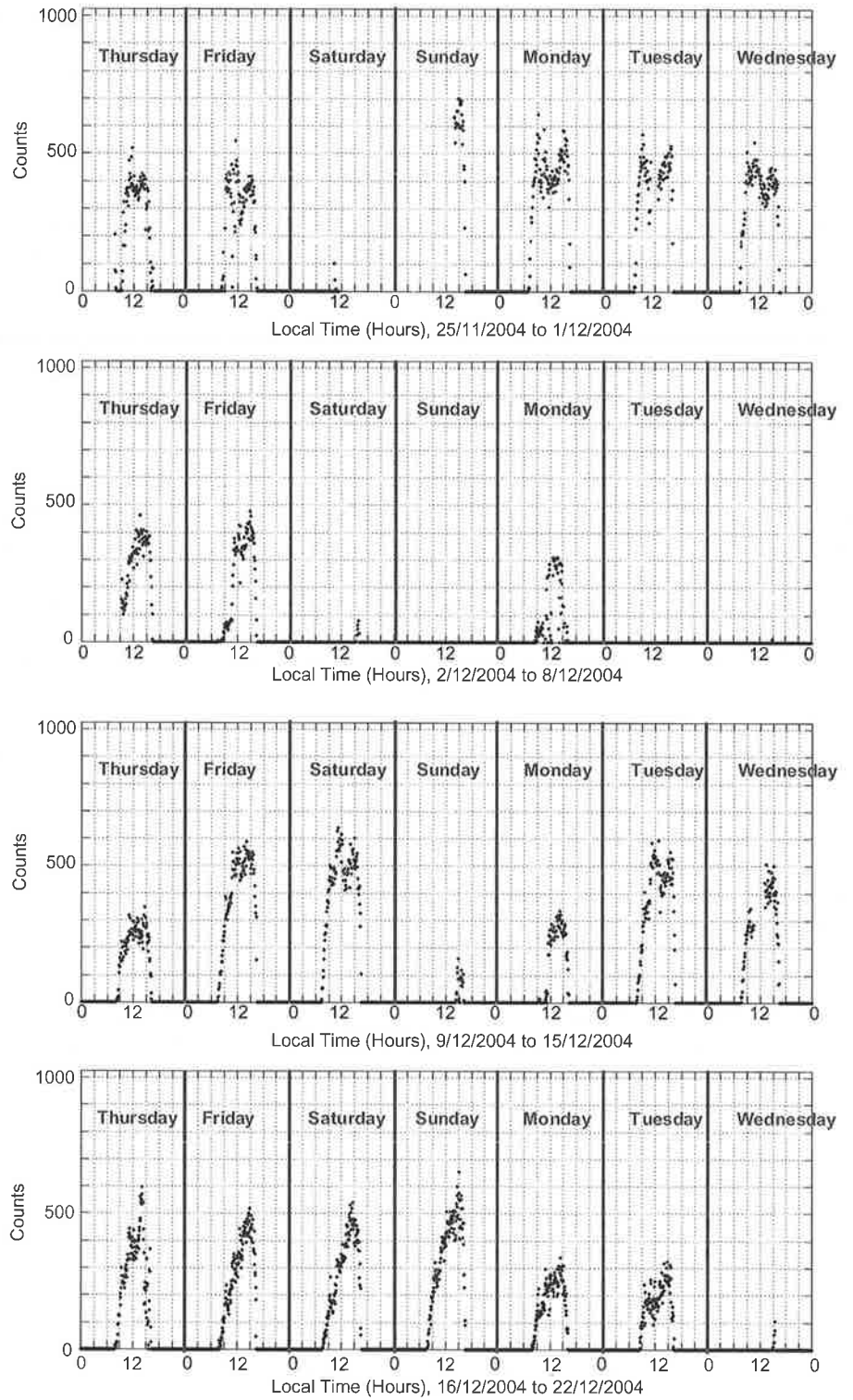


Fig. 5. Mean power levels.



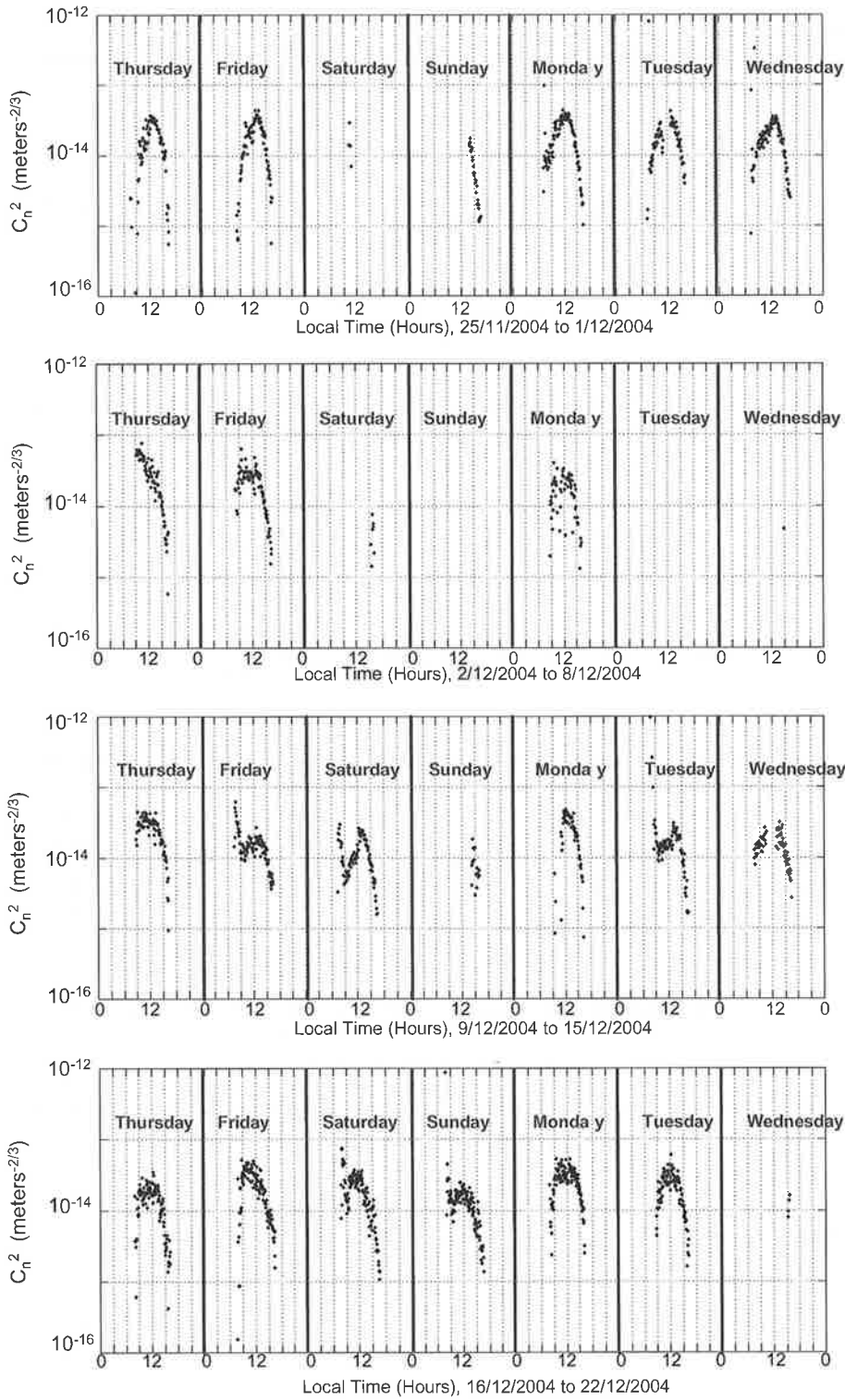
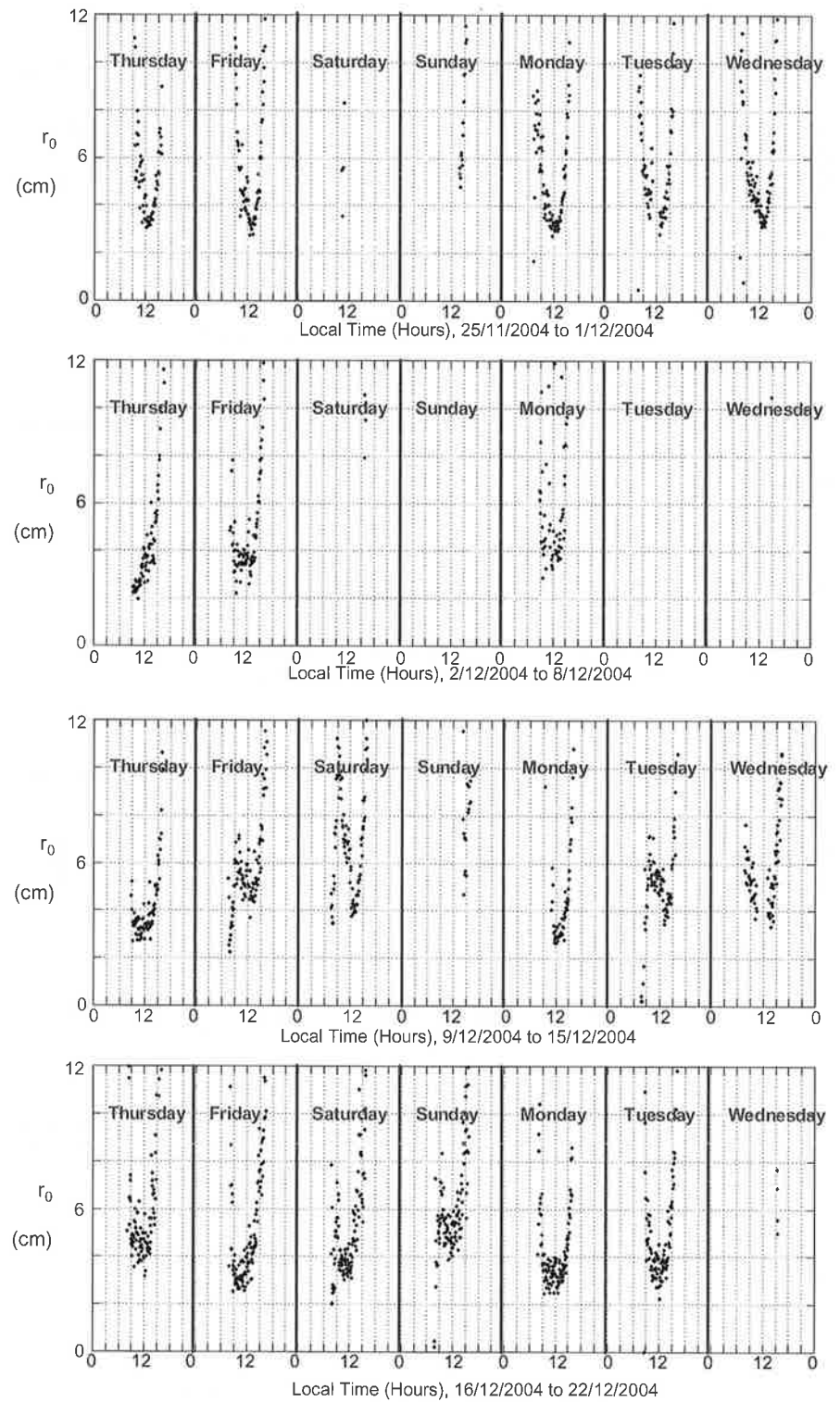


Fig. 6. Turbulence strength.



**Fig. 7.** Fried coherence length.

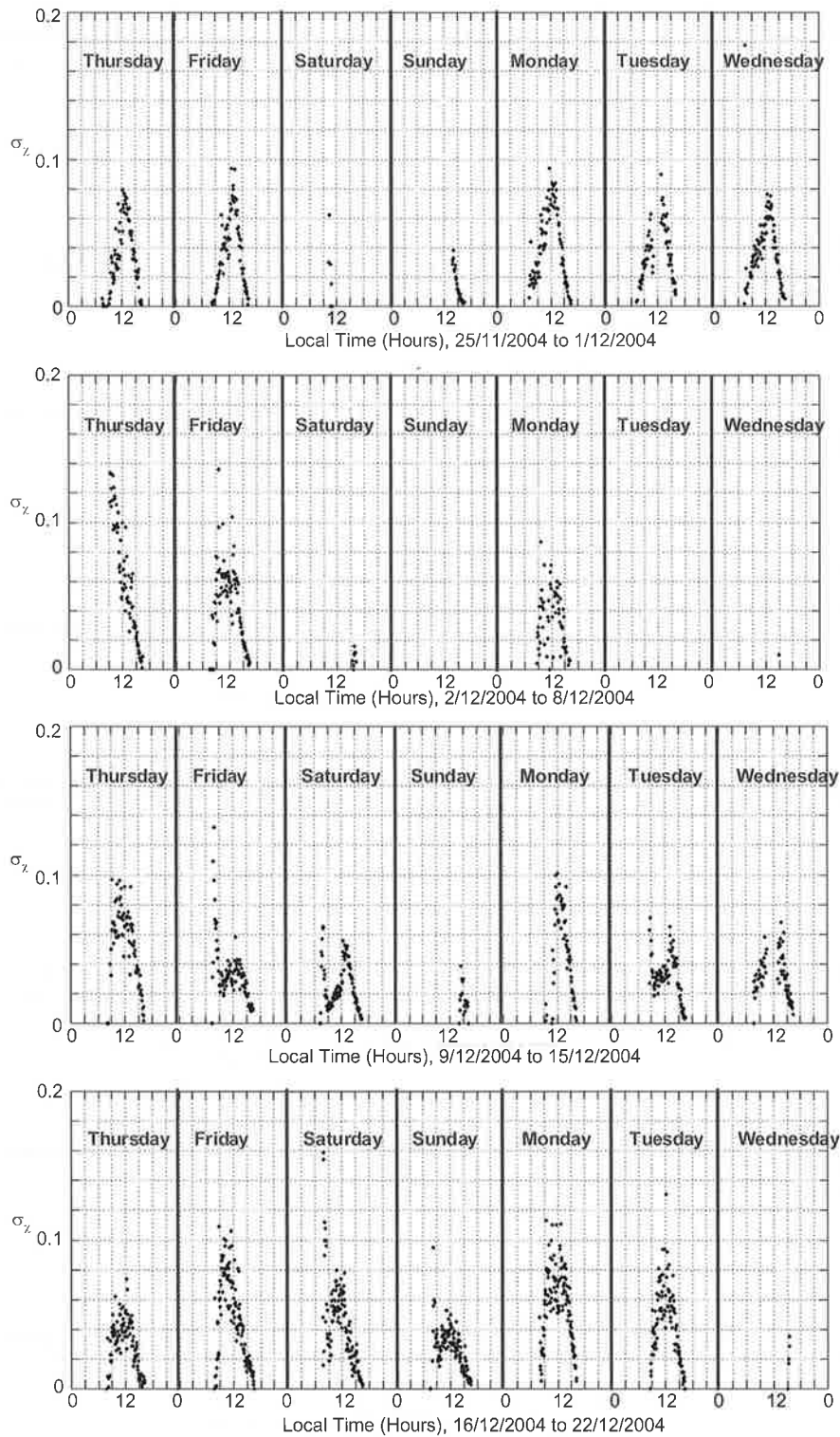


Fig. 8. Rytov number.

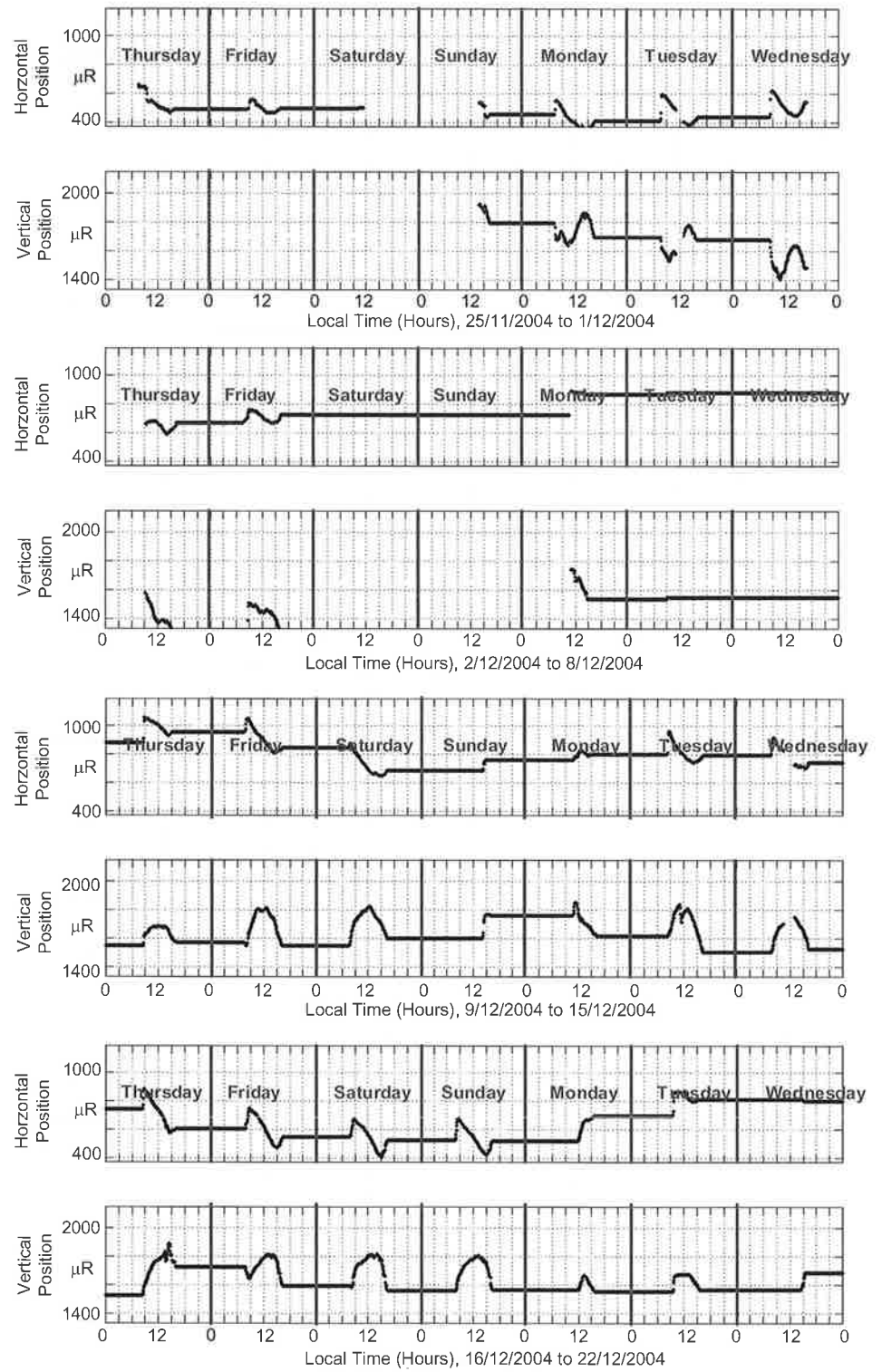


Fig. 9. Slow-building motion.

Figure 6 shows the mean calculated values for turbulence strength, using Eq. (1). Strength is generally worse (strongest) during midday, with morning and evening periods relatively calm. However, some mornings appear to begin with a period of relatively strong turbulence. On some days, we believe that the data show the morning “neutral event,” a short period of minimal turbulence that occurs when the air and ground temperatures are equal.<sup>1</sup>

There appears to be little, if any, difference in turbulence strength due to weekends or to the Thanksgiving or pre-Christmas holiday periods, suggesting that traffic in the intersection under the optical path, and other temporally changing man-made conditions, have little effect on the turbulence strength along the optical path.

Figure 7 shows the calculated Fried coherence length  $r_0$  for a spherical wavefront. The results suggest that it may be rather small during the day, 2.5 cm or so, and the AO design was modified to take this into account.

Figure 8 shows the calculated Rytov number for a spherical wavefront. The data generally show the Rytov number to be less than 0.1, and always less than 0.2, suggesting that we have relatively mild turbulence along the path. Under these conditions, the Rytov number should be identical to the log-amplitude variance of the wavefront at R1, something that is open to verification in future experiments.

Figure 9 shows the location of the upper-left-hand corner of the centroid region for the left-most glint on the focal plane image. During the experiment, the centroid regions were reset after each set of camera frames was read, in order to keep the glints approximately in the center of their respective regions. As can be seen, Fig. 9 is showing a slow drift that is likely due to uneven heating of portions of R1.

The absence of vertical data on some days is due to the curve's being out of range of the plot. These large jumps may actually be due to the telescope's being manually repositioned between data collection periods. However, the smaller continuous excursions are real and were taken into account in the AO design.

## 6. Summary and Conclusions

This experiment achieved its primary goal by giving us an indication of the turbulence strength along the optical path. The  $C_n^2$  values it provided are in the correct range to be believable and follow the same diurnal pattern noted by others who have measured turbulence strength along other optical paths.

The Fried coherence length, derived from  $C_n^2$ , suggests that we should be cautious about the actuator spacing in our AO system. We have, in fact, begun AO testing using a telescope aperture size reduced from the original 12-in. (0.3 m) diameter to a 4-in. (0.1 m) diameter.

The Rytov number, derived from  $C_n^2$ , suggests that we will see mild turbulence on this path, indicating that a single-stage AO design with a Hartmann wavefront sensor should do an adequate job correcting wavefront aberration.

Diurnal building motion, something we did not consider when we originally set up this experiment, is now seen to be relatively large and something that should be considered in the AO tilt mirror design.

There were also many shortcomings to this initial experiment, some anticipated, some not. Recognizing these is important, especially since we have the opportunity to overcome them with our next turbulence measurement experiment.

An obvious problem with the initial experiment is that we could collect data only during daylight when there were no heavy clouds or fog. We originally intended to use Christmas

lights on the sawhorse to collect data at night, but we found that the camera was not sensitive enough for this.

The camera we used in this experiment was not ideally suited for it, and one of the limitations imposed was our inability to collect frames fast enough to measure the Greenwood frequency, Tyler frequency, and other relevant temporal data relating to atmospheric turbulence.

Another problem with this experiment was a lack of independent verification of the data measured. There are several ways that the  $C_n^2$  values that we found could be wrong, one being the assumption that it is constant along the path. If the  $C_n^2$  values are wrong, the parameters useful for AO design and performance measurement, the Fried coherence parameter and Rytov number, would also be wrong.

The next turbulence measurement experiment should overcome all of these problems. We intend to use the same laser light that is collected by the 12-in. (0.3 m)-diameter AO telescope, split off a small portion in front of the AO, look at two 1-cm subapertures on either side of the split-off beam, and compute turbulence strength from the differential tilt of these two subapertures.

By using the laser light itself, we will eliminate our dependence on daylight and should be able to collect data around the clock on all but the most foggy days.

We have also purchased a new, more sensitive, camera that integrates the entire focal plane (or a selected region of interest) simultaneously and that has a fast enough readout to allow us to make temporal measurements of the turbulence.

The new turbulence measurement experiment will also sidestep the issue of independent verification of results. Rather than being interested in the turbulence strength itself, we are really interested in how the turbulence affects performance of the AO system.

By measuring the tilt between two subapertures in the AO system's main aperture, we are, in effect, measuring the very thing that the AO system is designed to correct. Our measurement of differential tilt can be independently checked, at any time we desire, by leaving the AO system in "open loop" (sending the deformable mirror to "flat") and measuring the differential tilt from subapertures of the wavefront sensor.

In our new experiment, we will also be able to compute the Rytov number from the differential tilt. This too can be independently checked by measuring the log-amplitude variance from the AO's wavefront sensor data. Since the initial experiment indicates that the turbulence is in the mild regime, Rytov number and log-amplitude variance should be in good agreement.

## References

<sup>1</sup>Beland, R.R., "Propagation Through Atmospheric Optical Turbulence," in *The Infrared & Electro-Optical Systems Handbook*, Vol 2, Infrared Information Analysis Center, Ann Arbor, MI, and SPIE Optical Engineering Press, Bellingham, WA, Chapter 2 (1993).

<sup>2</sup>Hutt, D. L., *Opt. Eng.* 38(8), 1288 (1999).

<sup>3</sup>Moore, C.I., H.R. Burris, M.F. Stell, L. Wasiczko, M.R. Suite, R. Mahon, W.S. Rabinovich, G.C. Gilbreath, and W.J. Scharpf, *Proc. SPIE* 5793, 78 (2005).

<sup>4</sup>Sasiela, R., *Electromagnetic Wave Propagation in Turbulence, Evaluation and Application of Mellin Transforms*, Springer-Verlag, Berlin (1994).

<sup>5</sup>Scintec AG, Product brochure for BLS-900 Scintillometer. <http://www.scintec.com/Site.1/PDFs/01.LayBLS.pdf>.

## The Authors

**Dr. David P. Dixon** is a senior member of the technical staff at Northrop Grumman Space Technology. He joined TRW in 1977 after completing graduate work in plasma physics at Columbia University. He joined Northrop when TRW was acquired in 2002. His first assignment at TRW was on an isotope separation project based on ion cyclotron resonances. Since then he has moved away from plasma physics, working on radar, passive millimeter wave imaging, and hyperspectral imaging in the visible and near IR. His recent interests have been in adaptive optics to compensate for distortion caused by atmospheric turbulence.

**Dr. Thomas C. Farrell** has been a practicing engineer for more than 20 years. In his first assignment in the U.S. Air Force, he was the on-site engineer at RAF Croughton, England, maintaining a variety of U.S. and NATO communications systems. Later he worked at Air Force Communications Command Headquarters, in Scott Air Force Base, Illinois, developing requirements and CONOPS and evaluating vendors' proposals, for military communications systems. He was with Northrop Grumman Space Technology (formerly TRW) for 11 years and worked most of that time on various programs associated with adaptive optics and atmospheric turbulence mitigation, for laser weapons and laser communications systems. Dr. Farrell is currently a U.S. government employee with Starfire Optical Range, Kirtland Air Force Base, New Mexico. He has a B.S. in electrical engineering from Lehigh University, an M.S. in electrical engineering from the Air Force Institute of Technology, and a Ph.D. in electrical engineering from the University of Kansas.

**Dr. Lee O. Heflinger** is a consultant for Northrop Grumman Space Technology. He joined TRW (then Space Technology Laboratories) in 1956 after completing graduate work in mathematics at UC Berkeley; he retired from TRW in 1991. During his career at TRW he worked on many aspects of spacecraft development, with an emphasis on optics and remote sensing. He played a key role in developing holography as a metrology technique to validate subsystem performance. He is the author of numerous papers and internal reports and holds several patents. Since retiring he continues to consult for NGST whenever they have an interesting problem.

**Mr. Stanley Klyza** holds an M.S. degree in electrical engineering from the University of Texas and was a member of the team designing and testing an adaptive optics beam correction system. He is currently a systems engineer with the James Webb Space Telescope program. Related experience in optical instrumentation includes spectrometers, ellipsometers, and biomedical applications.

**Mr. Kenneth J. Triebes** is the manager for beam control technology development as part of the Directed Energy Systems group in Northrop Grumman's Space Technology Sector. He has a Bachelor's degree from the Illinois Institute of Technology and has been working on the development and demonstration of directed energy weapons for more than 25 years. Since coming to Northrop in 2003, he has concentrated on the development of adaptive optical systems for atmospheric propagation compensation for tactical laser weapon systems. Prior to this, he enjoyed a 20-year tenure at Lockheed Martin Corporation, working on directed energy systems development, wavefront sensing and control, and advanced optical systems for science and defense applications.

# Kalman Estimation of Anisoplanatic Zernike Tilt

Todd M. Venema\* and Juan R. Vasquez<sup>1</sup>

Air Force Institute of Technology, 2950 Hobson Way,  
Wright-Patterson Air Force Base, Ohio 45433

*Anisoplanatism causes wavefront estimation errors when compensating for atmospheric turbulence of distant, fast-moving objects using wavefronts received from the object to measure the turbulence. An excellent example of this is the case of using adaptive optics for imaging or communication with satellites in low Earth orbit. By the time the light has made a round-trip from the satellite to the ground and back, the satellite will have moved approximately 50  $\mu$ rad. Linear estimation (extrapolation) of wavefront tilt parameters has been shown to mitigate anisoplanatism, providing significant improvement in a noise-free environment. We present Kalman filter estimation in lieu of simple linear estimation and demonstrate the robustness of this new approach.*

**KEYWORDS:** Anisoplanatic estimation, Kalman filter, Laser communication

## 1. Introduction

The goal of this paper is to demonstrate the effectiveness of using a Kalman filter to predict atmospheric tilts in a look-ahead situation. The research builds upon a technical note from David Fried, in which he uses a linear estimator to predict the look-ahead tilt needed for a ground-to-low-Earth-orbit (LEO) satellite system. Fried measures the tilt of a wavefront received from a target LEO satellite and computes an estimate of the tilt compensation needed to minimize atmospheric interference when communicating back to the satellite.<sup>2</sup> His estimate is based on the current tilt received plus the rate of change of that tilt multiplied by the amount of time it takes light to make a round-trip between the satellite and the ground. Rather than using simple linear estimation (i.e., extrapolation), this paper uses a Kalman filter to estimate the required tilt. Performance is measured by estimation error variance. The cases of no estimation, linear estimation, and Kalman estimation are compared.

A WaveTrain<sup>®</sup> simulation is developed that models a wavefront received from a point source on a LEO satellite. A noiseless sensor detects and records the light's complex field in the aperture plane. The simulations emulate conditions used by Fried and are used to develop time histories of Zernike tilt (ztilt) similar to his. Next, ztilt caused by atmospheric turbulence is modeled as a stochastic process so that a Kalman filter can track and estimate future ztilts. A key element of the modeling is matching the temporal bandwidth of the stochastic process with the temporal bandwidth of the effects from turbulence. Finally the

---

Received December 7, 2006; revision received August 21, 2007.

\*Corresponding author; e-mail: Todd.Venema@AFIT.edu.

<sup>1</sup>E-mail: Juan.Vasquez@AFIT.edu.